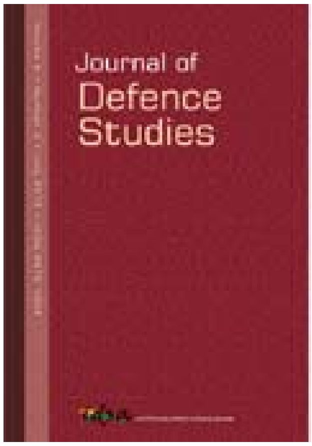

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Naval Operations Analysis in the Indian Ocean Region
A Review

*Arnab Das**

The end of the Cold War resulted in a fundamental swing from a navy designed to engage a blue water battle fleet to one focused on forward operations in littoral waters. The Cold War era had fuelled massive research and development (R&D) in design of sonars that was able to substantially minimize the uncertainties of the underwater environment. The shift of the naval theatre to the littoral waters led to a paradigm change in terms of technology requirements to retain the effectiveness of these sonars. The underwater environment in littoral waters is significantly influenced by the local conditions and is known to be site specific. The Indian Ocean Region (IOR) is even more challenging as the shallow waters are compounded by tropical conditions. This article identifies gaps in sonar technology contributing to their ineffectiveness and presents a naval operations analysis strategy to significantly improve their performance in the IOR.

INTRODUCTION

The success of any naval operation hinges predominantly on the performance of the sensors deployed and consistency of sensor

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performance. No naval commander can plan any operational deployment strategy if the uncertainties of system performance are high and very sensitive to the environmental fluctuations.[1,2] The issue of effectiveness could be addressed in two ways: One could be to have significant technology development to improve system effectiveness; and the second could be to evolve operational strategies that are capable of addressing the uncertainties of the system performance. In this article, I have attempted to examine both the approaches by ascertaining the impact of strategic shift of naval operations from deep waters to littoral waters.

The most critical aspect of any naval operation is early and reliable detection of the adversary. This is facilitated by deploying efficient sensors that are able to detect the adversary at long range, in spite of uncertainties of the underwater medium. The uncertainties of the medium in littoral waters originate due to proximity of the boundaries (surface and bottom) resulting in multiple interactions of the acoustic signal as it traverses from the source to the receiver.[3] These multiple interactions result in the acoustic signal getting modified, depending on specific characteristics of the bottom and the medium. In deep waters, these interactions are minimal and, thus, the site specific medium characteristics have less impact on the sonar performance. The tropical waters in the IOR additionally influence the acoustic propagation due to diurnal and seasonal temperature variations, and surface disturbances.[4]

The IOR has profound strategic relevance not only for the nations in the region but also for other major players of the world.[5] The bulk of the world's merchant fleet, including major petroleum exports originating from the Gulf, transits the Malacca Straits, the world's busiest sea lane, which encourages major powers of the world to maintain a strategic presence in the region. The present-day naval strategy is not so much on exercising sea denial but to maintain strategic presence, and switch to sea control whenever there is any threat to one's own maritime interest. This calls for comprehensive situational awareness and continuous monitoring, both on the surface and underwater, which again translates to deploying effective sensors. The geographical location of India in the IOR makes it a major player without choice. Further, in the recent past, the growing energy needs of China and the bulk of it transiting through the IOR has encouraged both China and the United States to ensure their strategic presence in the region.[6]

The Indian interest is not just restricted to ensuring territorial integrity of its close to 7,500 km of coastline, but also to utilize the vast resources in the IOR for its progress and development. In the present security dispensation, where subversive forces are also using the sea route, further developing effective underwater monitoring becomes an urgent necessity. Commercial off-the-shelf (COTS) systems no more serve the purpose as the Indian sub-continent presents its unique challenges that require custom-made solutions. The nation has to invest huge resources in oceanographic studies to develop a better understanding of the littorals around, to improve its own sonar performance, and deny the same to the adversaries.[7]

Having better sonar only addresses half the issue. Intelligent deployment of the sensors and the task force is another dimension that could offset some of the deficiencies of sonar effectiveness. Effective monitoring or search has two interrelated aspects—the first is reliable navigation, to have good area coverage without wastage of search effort, and the second is reliable detection at longer range. Both these aspects are highly sensitive to sonar performance in the presence of environmental degradation and severely restrict operational analysis efforts during tactical deployment of search resources. In a game with a theoretical approach for search of an underwater intruder, the operations analysis motivation is to maximize one's own effectiveness and deny the same to the adversary, or even to increase his uncertainties to put his tactical planning off gear.[8,9] In this article, a detailed study of the impact of a shift of the naval battlefield from blue waters to littorals is presented. The way ahead for improved sensor performance is discussed more from the operational analysis standpoint to exploit the medium. Finally, an operational analysis assessment is presented to put in perspective the technology gap, the tactical deployment aspect, and how to evolve the operations analysis strategy in the Indian context.

The article has been organized into six sections. The first section presents the characteristics of propagation of sound in the ocean medium. The second section discusses the basics of sonar theory relevant to the search problem. The third section elaborates the aspects of littoral waters that impacts sonar performance compared to the deep waters. The fourth section enumerates the basics of search relevant to operations analysis. The fifth section presents the operations analysis strategy applicable to the Indian context while the last section concludes the findings.

PROPAGATION IN THE OCEAN MEDIUM

The characteristics of the water medium only allows pressure (acoustic) waves to propagate, unlike the air medium where electro-magnetic waves propagate at the speed of light (i.e. 3×10^8 m/s). However, the acoustic waves are highly dependent on the medium properties as the propagation takes place by transferring energy from one molecule to another, and so on. Thus, the density of the medium determines the speed of propagation of the wave in the medium. To present the order of such dependence, we may take the example of the speed of sound: in air, it is 345 m/s, in sea water 1500 m/s, and in steel 5800 m/s. Any change in the medium density due to temperature variation or contamination, thus, changes the speed of sound. Typically, the speed of sound in the ocean depends upon three factors—temperature, salinity, and depth, and varies as given below:[10,11]

Temperature : + 2.743 m/sec per °C increase in water temperature.
Depth : + 0.016 m/sec per meter of depth.
Salinity : + 1.21 m/sec per ppt increase in salinity.

The information in the signal is packed by varying the frequency. We are aware that the frequency 'f', depends on the wavelength 'λ' and the speed of sound 'c' in the medium given by the relation $c = f\lambda$. Thus, it is evident that the different frequency components of the signal travel at different speeds and, also, the same frequency component may travel at different speeds if the speed of sound in the medium changes due to fluctuations in the medium properties. This translates to the signal travelling through multiple paths from the source, combining at the receiver with different phases due to a varying time delay. The multi-path arrivals at the receiver, thus, combine non-coherently to result in signal distortions due to random medium fluctuations. The received signal thus arrives with random amplitude, phase and frequency. Figure 1 depicts a typical sonar deployment scenario for detection of an underwater target.

The underwater channel also displays a very unique band limited property not observed in any other medium.[12] The fact that the signal gets influenced by the interaction with the medium impurities and boundaries whenever the size of these become comparable to the wavelength of the propagating signal, causes this band limited behaviour. From the relation presented above, we know that for c = 1500 m/s and f = 10 kHz, the wavelength is λ = 15 cm. It is easily comparable to the surface roughness of the sea surface waves and the bottom undulations that result in

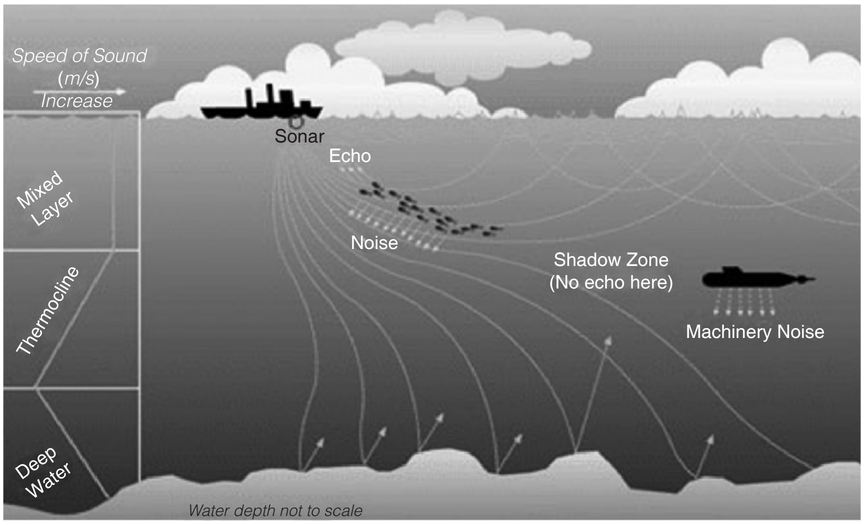

Figure 1 Detection of an Underwater Target by Sonar

scattering of the incident sound waves and also poor reflection from these boundaries. Further, the size of impurities in the ocean medium, including fishes and dissolved particles, result in enhanced absorption of the acoustic signal at higher frequencies. Some measure of absorption loss in the ocean medium due to frequency and depth is exemplified by the following data:

Increases very rapidly at higher frequencies
    0.05 dB/km at 1 kHz
    0.5 dB/km at 10 kHz
    25 dB/km at 100 kHz
Reduces with depth
    2% reduction every 300m increase in depth

The ocean medium presents a layered structure speed due to varying medium characteristics depending upon the temperature and the depth.[13] In the deep ocean, these are typically divided into four main layers as shown in Figure 2 and enumerated below.

(a) *Surface Layer:* This is the top layer that is sensitive to the diurnal temperature variations and normally presents minimal variation in sound speed with depth due to isothermal behaviour on account of churning of the water because of surface disturbances.

(b) *Seasonal Thermocline:* This presents decrease in speed of sound due to decrease in temperature with depth, the temperature being the dominant factor. Mean temperature gradient depends on seasonal factors; hence, the name.

(c) *Main Thermocline:* This presents decrease in speed of sound due to decrease in temperature with depth. Mean temperature gradient depends on the location with respect to the poles or the equator. Tropical, temperate and polar regions have a different structure.

(d) *Deep Isothermal Layer:* This layer is independent of the temperature variation; hence, the name. The temperature of the water remains more or less constant at 4°C throughout the layer. The speed of sound increases with depth. The deep isothermal layer is independent of diurnal and seasonal variations and is invariant to the location on earth.

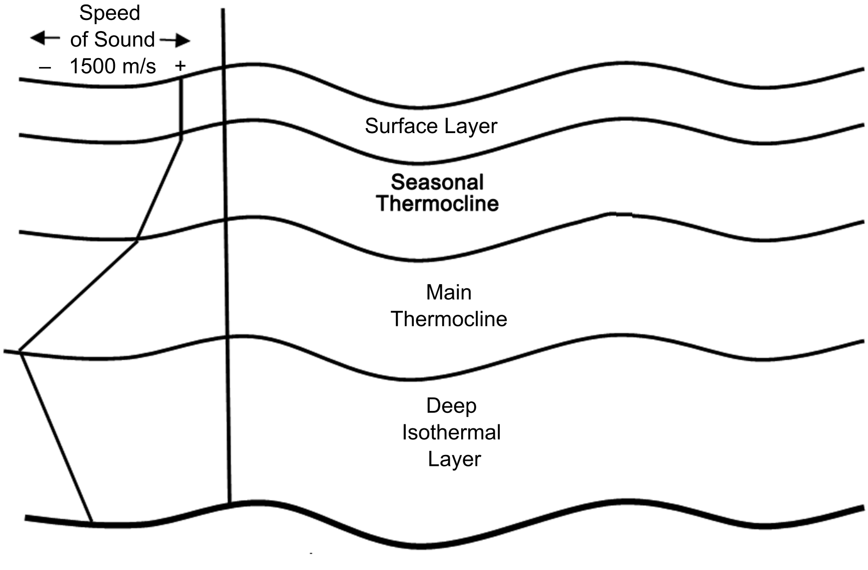

Figure 2 Sound Velocity Profile in the Deep Ocean

An important aspect of sound propagation in the ocean medium due to its layered structure is refraction, and from Snell's law we know that the angle of refraction is dependent on the instantaneous speed of sound in the layer.[14] Thus, due to the variation in the speed of sound at varying depths and also fluctuations due to temperature and other factors, the signals arriving at the distant receiver gets randomly modified. Figure 3 presents the refraction of acoustic signal in the ocean medium governed by Snell's law.

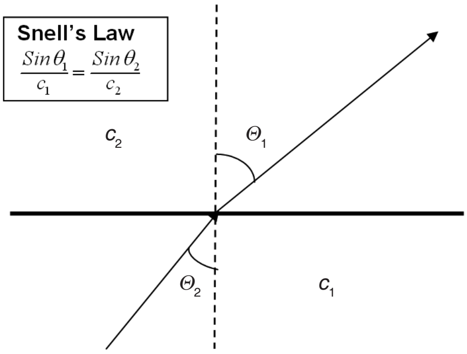

Figure 3 Refraction of Sound in the Ocean Medium

It may be noted that the point of minimal sound speed generates an imaginary sound axis that ensures focussing of the sound around the axis due to refraction of the sound during propagation towards the axis on either side (see Figure 4). In case there is sufficient depth above and below this sound axis, the sound may propagate without any surface and bottom reflection resulting in minimal attenuations on account of absorption losses at the boundaries. Such a duct formation is also known as SOFAR channel. The SOFAR channel facilitates long range propagation and numerous sonar deployment strategies take advantage of such a channel to maximize the sonar range.[15]

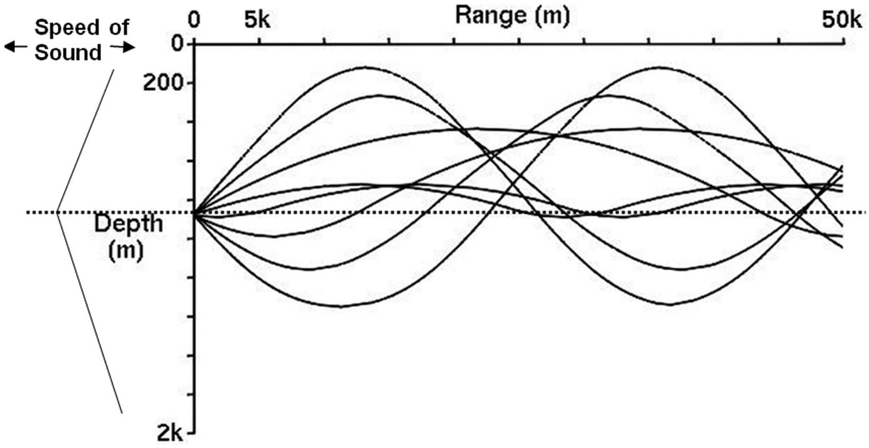

Figure 4 Propagation of Sound Around the Sound Axis

Alternately, we could also have a situation wherein the depth of water is not sufficient and the sound propagation results in multiple sea surface and bottom interactions resulting in high attenuations of the sound signal, thereby reducing the sonar range. Such propagation is pictorially represented in Figure 5. This could arise due to two factors: the first is that the depth of water is not sufficient to allow refraction of the sound prior to boundary interactions; second, the sound speed profile in the layered structure could be such that the sound axis may not exist within the depth.[16]

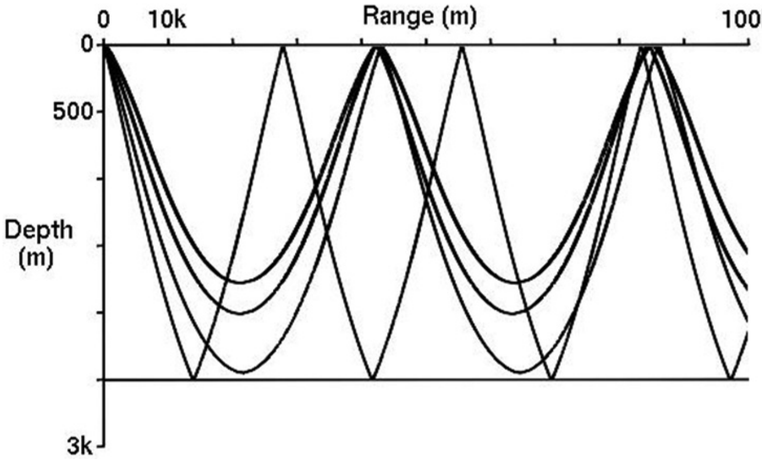

Figure 5 Propagation of Sound with Multiple Boundary Interactions

The propagation of sound in deep waters and shallow waters are different due to the variability of the layers and the sound velocity profile as presented in Figure 2. There are two definitions of shallow waters—hypsometric and acoustic. The hypsometric definition is based on the fact that most continents have continental shelves bordered by the 200 m contour line, beyond which the bottom generally falls off rapidly into deep water. Therefore, shallow water is taken to mean continental shelf waters shallower than 200 m. Acoustically, shallow water conditions exist whenever the propagation is characterized by numerous encounters with both the sea surface and the sea floor. It is possible that an ocean region could be hypsometrically deep; however, due to a certain sound velocity profile, there are multiple reflections from the sea surface and the sea bottom that makes it acoustically shallow and vice-versa.

The above discussions very clearly enumerate that the propagation of sound in deep waters is very conducive to long-range propagation with minimal distortions and attenuations. However, in the littoral waters, the propagation is highly restricted due to multiple bounces from the sea surface and the sea bottom. The depth of water available for propagation also determines the wavelength of the signal that can be transmitted, which explains the band limited nature of the shallow water channel. The limitation of the littoral waters is discussed in a subsequent section.

SONAR SYSTEM

Sonar systems are primarily divided into two types based on functionality. The active sonar wherein a signal is transmitted by the sonar projector that illuminates the target and the reflections (or echo) from the target arriving at the receiver are analysed to draw conclusions. In a tactical situation, where stealth becomes the key to survival, transmission could give away one's own position in a hostile environment. Therefore, passive sonars were developed that would remain passive and attempt to receive emissions from the target and analyse them to detect and classify the contact. The sonar equation is a tool to comprehensively evaluate various parameters that influence sonar performance.[17] The relevance of the sonar equation can be explained as given below:

(a) Predict SONAR performance, e.g., what is the achievable range in specified conditions?
(b) Verify existing SONAR design, e.g., will the SONAR perform as specified?
(c) Assist in SONAR design, e.g., what should be array dimension?

The parameters that are combined together in a sonar equation can be broadly classified into three categories:[18]

(a) Equipment
    (i) Source Level (SL): The signal intensity at the source location in the direction of interest.
    (ii) Directivity Index (DI): Ability of Sonar to concentrate the receiver beam (and Transmitted beam).
    (iii) Self Noise Level (NL): Noise intensity generated at the Sonar due to own ship.
    (iv) Detection Threshold (DT): Signal-to-Noise ratio required for specified performance.
(b) Environment
    (i) Transmission Loss (TL): Loss in signal intensity due to propagation in water.

  - (ii) Ambient Noise level (NL): Interfering signal other than the signal of interest.
  - (iii) Reverberation Level (RL): The interfering signal due to reflection of own transmission from boundaries and medium impurities other that the desired target echo.
- (c) Target
  - (i) Target Strength (TS): The echo return from the target, depending on the size and characteristics of the target material.
  - (ii) Radiated Noise Source Level (SL): In case of passive sonar, the intensity of the emissions from the target.

Based on the above parameters we now state the sonar equations:

Active Sonar (Noise limited)

$$SL - 2TL + TS = NL - DI + DT$$

Active Sonar (Reverberation limited)

$$SL - 2TL + Ts = RL + DTR$$

Passive Sonar

$$SL - TL = NL - DI + DT$$

Certain derived parameters are presented that are relevant to the naval operations analysis applications:

Echo Level = SL – 2TL + TS dB
Noise Masking Level (NML) = NL – DI + DT dB
Reverberation Masking Level (RML) = RL + DTR dB
Figure of Merit (FOM) = SL – (NL – DI + DT) dB

FOM is the maximum allowable one-way transmission loss in Passive Sonar or maximum allowable two-way loss for TS = 0 dB in Active Sonar.

In addition, it is important to elaborate on the sonar parameters that will be impacted by the littoral operation. The littoral waters will impact the environmental parameters as listed above and we will take up these parameters one by one to elaborate on the specifics of littoral waters.[19]

Transmission Loss

The transmission loss (TL) comprises two parts, namely, the spreading loss and the absorption loss. The spreading loss is due to the spread of the signal in the three-dimensional space away from the source. In the deep waters with minimal influence of the boundary conditions, the spreading will follow an inverse square law also known as spherical spreading. In shallow waters, due to the influence of the sea surface and the bottom, the spreading will be restricted in two dimensions and would follow cylindrical spreading. However, it may be noted that the spreading is also accompanied by absorption at the sea surface and the sea bottom, depending upon the nature of the boundary. Spreading loss is frequency independent. The absorption loss is on account of the absorption of sound due to interaction of the sound waves with numerous impurities, and the surface and bottom roughness. This is related to scattering and diffusion of sound due to this interaction whenever the size of the interacting object is comparable to the wavelength of the sound signal. In the littoral waters, the surface and the bottom interactions are significant, so the greater influence of TL. Also, the shallow waters that comprise 7.5 per cent of the total world oceans contain close to 90 per cent of marine life. This translates to higher impurities that will enhance the interaction and consequent losses.

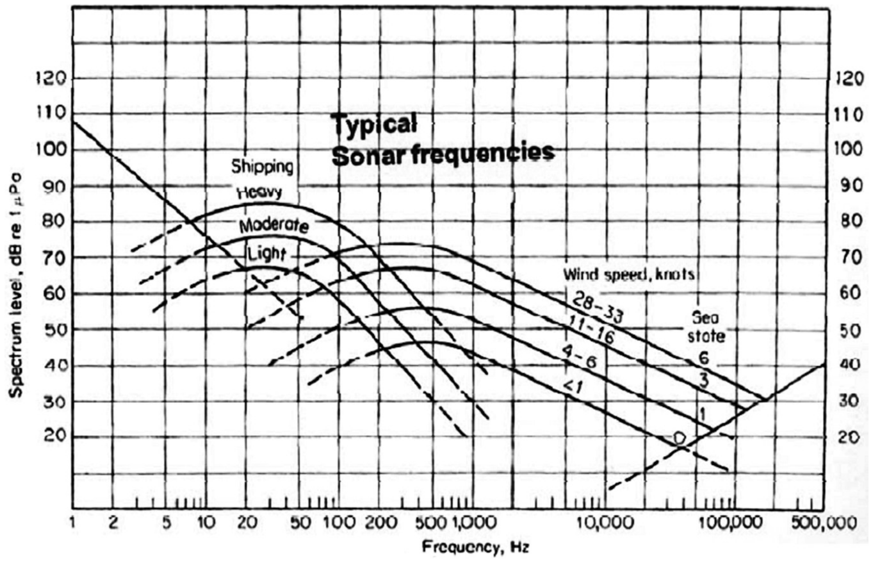

Figure 6 Ambient Noise in the Ocean[20]

Ambient Noise

The background noise present in the ocean, or ambient noise, has many different sources and varies with location and frequency. At the lowest frequencies, from about 0.1 Hz to 10 Hz, ocean turbulence and microseisms are the primary contributors to the noise background.

Typical noise spectrum levels decrease with increasing frequency from about 140 dB re 1 µPa²/Hz at 1 Hz to about 30 dB re 1 µPa²/Hz at 100 kHz. Distant ship traffic is one of the dominant noise sources in most areas for frequencies of around 100 Hz, while wind-induced surface noise is the main source between 1 kHz and 30 kHz. At very high frequencies, above 100 kHz, thermal noise of water molecules begins to dominate. The thermal noise spectral level at 100 kHz is 25 dB re 1 µPa²/Hz. The spectral density of thermal noise increases by 20 dB per decade (approximately 6 dB per octave). Transient sound sources also contribute to ambient noise. These can include intermittent geological activity, such as earthquakes and underwater volcanoes, rainfall on the surface, and biological activity (biological sources include cetaceans [especially blue, fin and sperm whales], certain types of fish, and snapping shrimp). Figure 6 presents the ambient noise levels in a pictorial form.

Reverberation

The reverberation, in case of an active sonar, depends on the scatterer in the immediate vicinity of the projector. There are three types of reverberations—volume, surface, and bottom as presented in Figure 7. The volume reverberation depends on the type of impurities in the immediate vicinity; thus in shallow waters where the marine life is likely to be high, it will translate to higher levels of reverberation. In shallow waters, the bottom type will be influenced by local conditions like river mouth, etc., that may be muddy bottom or rocky bottomed. The sea surface roughness will again be impacted by the wind and sea state locally.

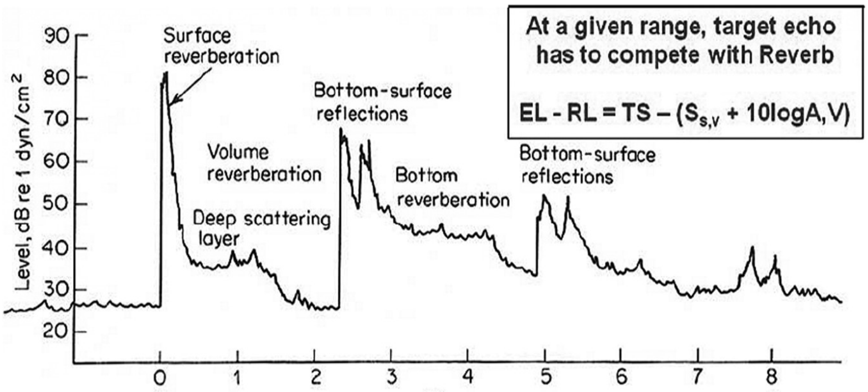

Figure 7 Reverberation Loss for Explosive Charge[21]

At the sonar receiver, the received echo will have to compete with the reverberation level, particularly at short ranges.

Thus, we observe that the medium characteristics introduce enormous amount of uncertainties in sonar performance. The way ahead is to invest efforts in oceanographic research to understand the propagation characteristics and ambient noise behaviour, and create predictive models to connect the measurable physical parameters to the realistic oceanographic data.[22] This will facilitate design of online adaptive inversion and filtering algorithms, to minimize the fluctuation due to shallow underwater channel distortions and the ambient noise variability, based on measureable physical parameters like instantaneous sea state, bottom type, bottom profile, wind speed, sound velocity profile, and sea surface temperature, etc. Tropical waters present significant fluctuations within these parameters on a diurnal basis that can impact sonar performance. Research literature presents numerous adaptive algorithms that can improve sonar performance, given the instantaneous physical parameters at the deployment site. Thus, effort is required to generate the predictive models and subsequent validation of such models with extensive at sea oceanographic experiments in the littorals within the immediate vicinity.[23]

Littoral Waters

The littorals of the world are drastically different from the open ocean, but more so than with regard to the physical features. The littorals are close to shore and this causes an increase in environmental noise due to waves breaking on the shore, waves crashing on man-made structures, surface noise from waves reflecting off the bottom, and the passing of commercial and recreational craft. This causes an increase in the environmental noise that the singular signal emitted by the submarine must be detected in. The comparative shallowness of the ocean bottom creates a high incidence of bottom reflection that attenuates any signal from a submarine at a much faster rate than the open ocean, where the signal can travel for miles before interacting and being absorbed by the bottom or any other object. Conversely, any active sonar signal employed to detect a submarine operating in the littorals would suffer from the same rapid degradation in signal strength, resulting in a limited range and effectiveness of active sonar.[24]

The littorals also possess the unique characteristic of mixing fresh and salt water at the mouth of rivers and streams that empty into the ocean.

This mix of fresh and salt water creates a dynamic salinity that affects the speed of sound travelling in water, causing it to reflect away from changes in salinity much the same way that it reflects away from temperature changes. There is also a temperature change that often accompanies these salinity changes, magnifying the effect that mixing sources of water has on sound propagation.

Geospatial concerns in the littorals are related to the effects of the topography of the bottom. There has already been a brief discussion of the effect that a shallow bottom has on sound propagation, but that is not the sole concern of geospatial effects. Differences in bottom make-up, such as clay or soft sand, will affect how much, or how little, sound reflects off the bottom. In a sandy bottom, a submarine may make a noise and that noise will never be heard because the bottom, instead of reflecting the sound outward, absorbs it. While this presents a difficulty in detecting submarines, it is something that could be accounted for if the make-up of the bottom is known.

Marine life is sparse in the open ocean, deep-water environment of the Cold War Anti-submarine Warfare (ASW) problem. By necessity, marine life lives closer to shore, in the shallow, warm water of the littorals. This presents another series of challenges to the ASW problem, aside from the numerous issues regarding the use of active sonar and its possible effect on marine mammals. Chief among these is the effect on environmental noise caused by marine life. Simply put, almost every living creature makes some sort of noise either for communication or simply through the act of living. Every sound adds to the total level of background noise the ASW force must now try to detect the quiet submarine signal through.

The tropical conditions further complicate acoustic signal propagation in the ocean medium due to variability in the temperature over the day and seasons. While these acoustic conditions make sound-based ASW difficult to execute, they also make the planning nearly impossible to optimize. Suppose an area search requires sensors placed in a geometric pattern based on the expected detection range (which can be predicted based on water depth, temperature, salinity, wave action, shipping density and a number of other factors). Further, suppose that there are considerable costs involved with the placement of the sensors—either the cost of the sensor or the cost of deployment, or perhaps both. In a resource constrained world, efficient force deployment is called for and likely draws from a predetermined inventory that was predicated on expected requirements. Now suppose the detection ranges can vary by a factor of 10 over the course of a single deployment or in the contemplation of deployment to different locales—what is the properly sized force for such uncertainty? Availability of assets quickly becomes a direct result of strategic decision-making.

SEARCH AND PATROL

The most important instrument/equipment in any search operation is the sensor and the sensor parameter relevant for such an operation is the detection range $\boldsymbol{R}$. If the relative motion between sensor and target is assumed to be at constant, non-zero velocity in two dimensions over a complete line, the lateral range is the distance between searcher and the target at the point where the distance is minimal, termed as the closest point of approach (CPA), as shown in Figure 8(a). The lateral range is taken as positive if the target lies to the left of the searcher at the CPA or otherwise negative.[25]

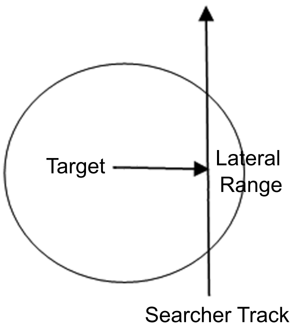

Figure 8(a) Top View of the Basic Experiment

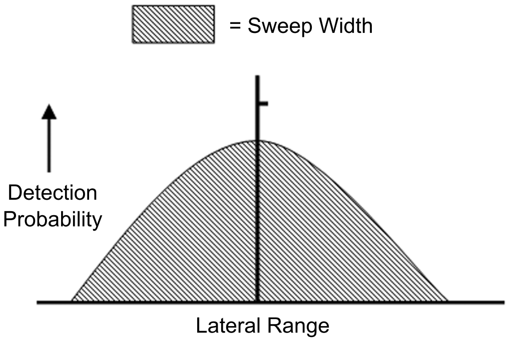

Figure 8(b) Lateral Range Curve

It is important to note that the detection may or may not happen at the CPA, it is likely to happen anywhere inside the sensor range before or after the CPA. The lateral range $\boldsymbol{r}$ could vary between $-\boldsymbol{R}$ to $+\boldsymbol{R}$. Let $P(\boldsymbol{r})$ be the detection probability at the lateral range. A graph of detection probability $P(\boldsymbol{r})$ versus lateral range $\boldsymbol{r}$ is a lateral range curve as shown in Figure 8b. The area under the lateral range curve is called the sweep width $\boldsymbol{W}$. It may be noted that the sweep width has dimension of length, in spite of being an 'area,' since the vertical dimension of a lateral range curve is dimensionless. The location of the target being unknown, thus, the lateral range itself will be random and let the density function for the lateral range be $f(\boldsymbol{r})$ for some pass. Thus, the probability of detection $\boldsymbol{P_D}$ for that pass could be obtained by averaging over the lateral range as given below:

$$P_D = \int_{-\infty}^{\infty} f(r)\,P(r)\,dr \cong f(0)\int_{-\infty}^{\infty} P(r)\,dr = f(0)W \qquad ...(1)$$

Search in a fixed region of area ***A*** for an assumption of single stationary target, without any prior knowledge of the target location, is considered here which translates to the target's location density being constant within the region, necessarily 1/***A***. The single searcher looks continuously for the target, moving at speed ***V*** and with fixed detection radius ***R***. We assume a cookie cutter sensor meaning ***W*** = 2***R***. After a time ***t***, the searcher covers a region of length ***Vt*** and width ***W***, the area of which is ***VWt***. The coverage ratio ***z*** is the ratio ***VWt/A***, the ratio of the area covered to the area of uncertainty. Typically, the search continues till the time detection occurs and, thus, the detection probability is represented as a function of time $\boldsymbol{P_D(t)}$. If ***T***, is the random time until the first detection occurs, Since $\boldsymbol{P_D(t)} = \boldsymbol{P_D(T \leq t)}$, the mean value of ***T*** is given by:

$$E(T) = \int_{0}^{\infty} P(T \succ t)\,dt = \int_{0}^{\infty} \left(1 - P_D(t)\right)dt \qquad ...(2)$$

The assumption that the area ***VWt***, is all located within the region and, also, that there is no overlap with itself, leads us to the relation for the detection probability:

$$P_D(t) = \min(1,\ z),\ \text{where, } z = {}^{VWt}\!/_{A} \qquad ...(3)$$

Search in a given area could be undertaken in three different plans as presented in literature. These are exhaustive search, random search, and the inverse cube search. The **exhaustive** search follows a path that looks something like the path of a lawn mower or a spiral path to cover all of a circular area without overlap. The maximum time to detect is ***A***/(***VW***) in an exhaustive search and the mean time to detection is half of that.

The **random** search assumes a random path without any predetermined plan. The default way to implement a random search is widely known as 'diffuse reflection', motivated by the path of a light photon making diffuse reflection from a rough wall. The overall detection probability of a random search is given by:

$$P_D = 1 - \left(1 - {}^{VWt}\!/_{nA}\right)^n \cong 1 - \exp(-z),\ \text{where, } z = {}^{VWt}\!/_{A} \qquad ...(4)$$

The above approximation is valid for vary large value of *n*, which symbolizes the entire area split into *n* small partitions, with a chance of ***VWt/nA*** of covering the target. When ***t*** is very small, the coefficient (***VWt/A***) can be equated to the constant detection rate λ in a Poisson process of detections, with $P_D(t)$ being the probability that there are no detections over the interval of length ***t***. Thus, in a random search, the time ***T*** to detection is an exponential random variable with parameter λ, and its mean is therefore (***A/VW***), the reciprocal of λ.

The inverse cube law of search assumes that the target detection is a Poisson process where the event rate is proportional to the solid angle, with the proportionality constant depending on things like the target size, its contrast with the background, and other parameters. The sensor sweep width depends on the proportionality constant. Considering a lawn mower search of an area, where the searcher's parallel passes are separated by a track spacing of ***S***, the resulting coverage ratio is ***W/S***, and the detection probability is given by:

$$P_D(t) = 2\Phi\left(z\sqrt{\frac{\pi}{2}}\right) - 1, \text{ where, } z = {}^{W}\!/_{S} \qquad \ldots(5)$$

where Φ() is a cumulative normal distribution. Since the track spacing is possible when searching a fixed area ***A*** at speed ***V*** in time ***t*** is ***S***=***A***/(***Vt***), the definition of ***z*** is the same as the coverage factor.

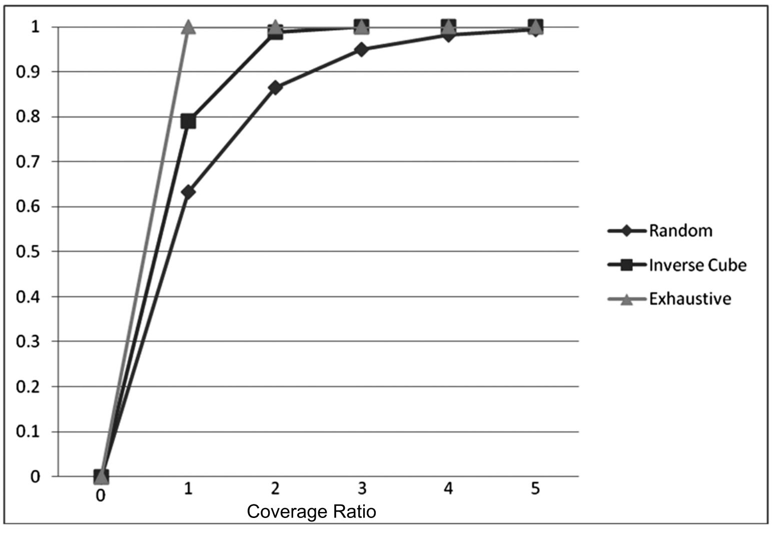

Figure 9 Detection Probability versus Coverage Ratio

Source: Washburn and Kress, *Combat Modelling*, n. 2.

Figure 9 represents the detection probability versus coverage ratio for the three search plans. It is apparent that all the three curves start with the same slope (namely λ), but otherwise differ significantly for coverage ratio 1.

Moving Targets

If the target moves 'at random' within the search area at speed U, then it is reasonable to suppose that the angle θ between the target's velocity and the searcher's velocity is equally likely to be anything between 0 and 2π. If the searcher's speed is V, then the relative speed between the two is $\sqrt{U^2+V^2-2UV\cos(\theta)}$. The average relative speed is, therefore:

$$\tilde{V}=\int_0^{2\pi}\sqrt{U^2+V^2-2UV\cos(\theta)}\frac{d\theta}{2\pi} \qquad \text{...(6)}$$

The quantity $\tilde{V}$ is sometimes taken to be a 'dynamically enhanced' search speed in the sense that a searcher with speed $\tilde{V}$ will have the same prospects of finding a stationary target as would a searcher with speed V looking for a target moving at speed U. The equation (6) is a symmetric function of V and U, thus it makes no difference even if the two are interchanged. Although there is no closed form for the integral in equation (6) (it is a complete elliptic integral of the second kind), we can at least give some upper and lower bounds on $\tilde{V}$:

$$\max\,(U,V)\le\tilde{V}\le\sqrt{U^2+V^2} \qquad \text{...(7)}$$

The bounds are closest to each other when U and V differ significantly. The worst case is when U and V are equal (to 1, say), in which case the lower bound is 1, the upper bound is 1.41, and $\tilde{V}$ is actually $4/\pi = 1.27$.

Evasive Targets

The success of a search operation for a moving target depends on the target's motivation. Three clear cases can be distinguished:

(a) The target may desire detection. It is important to the search and rescue community, but we will not delve into it here.
(b) The target may be indifferent to detection or unaware that it is being searched for.
(c) The target may know that it is being searched for and take measures to avoid being found. This is often the appropriate

assumption in combat models. This kind of evasive target is the current subject.

The simplest case is to imagine that the searcher and target take turns acting. First, the searcher distributes some effort over the cells. If search is unsuccessful, the target may move to a different cell, after which the searcher tries again, and so on. It is not usually realistic to suppose that the target's next position is independent of its current position, so we suppose instead that the target's motion is a Markov chain.[26] This requires that we specify the chain's transition function, in addition to the probability distribution of the target's initial position. For a Markov chain defined on a set of cells C, the *transition function* $\Gamma(x, y, t)$ is the probability that a target in cell x at time t will move to cell y at time $t + 1$. Such a function must be nonnegative, summing on y to 1 for each $x \in C$, and also for each time t.

Let $P(x, t)$ be the probability that the target is located in cell x at time t and is not detected by any of the searches before time t. If normalized to sum on x to 1, this function is the one that a searcher wondering where to look at time t would call 'the current distribution of the target's location' and might wish to see displayed to guide his search at time t. Let $q(x, t)$ be the non-detection probability for a look at time t, given that the target is in cell x. This function is determined by the searcher at time t when he decides which cell to look in or possibly how to spread his effort over multiple cells. The formula that advances time is then given by the theorem of total probability:

$$P(y,t+1) = \sum_{x \in C} P(x,t)\,q(x,t)\,\Gamma(x,y,t);\ y \in C \qquad \ldots(8)$$

The only way to get to cell y at time $t + 1$, without being detected before $t + 1$, is to be in some cell x at time t, without being detected before t, to not be detected at time t, and to move from x to y. That statement is formalized by equation (8). There is a technique—the FAB (Forward and Backward) algorithm—designed to find the globally optimal distribution of effort when the target moves.

An abstract version with a pursuer and evader has five parameters:

(a) U is the evader's maximum speed (we assume unlimited endurance).
(b) V is the pursuer's speed.
(c) W is the pursuer's sweep width.

(d) t is the time after the initiating event when the pursuer arrives at the datum, the 'time late.'
(e) t is the amount of time that the pursuer spends searching.

The exact solution of this problem as a game is again unknown, but, if we assume that the pursuer at all times searches randomly within the gradually expanding circle that represents the evader's farthest distance from the datum, the detection probability can be shown to be:

$$P_D(t) = 1 - \exp\left(-\frac{VW}{\pi U^2}\left(\frac{1}{\tau} - \frac{1}{\tau + 1}\right)\right) \qquad \text{...(9)}$$

In this case, results are very sensitive to the evader's speed, which is squared in equation (9). The evader can use his speed effectively to become more and more lost in the two-dimensional plane.[27]

Barrier Patrol

Searchers sometimes take advantage of environmental features that force moving targets through narrow restrictions. These restrictions might be straits or harbour mouths at sea. By remaining in the vicinity of these restrictions, the searcher hopes to construct a one-dimensional barrier to movement, rather than deal with a large, two-dimensional expanse. We assume that the searcher uses a cookie-cutter sensor with definite range R.

Let L be the width of the barrier to be protected. We assume that target motion is perpendicular to the barrier. Consider a target whose speed is U and assume that the barrier penetration point is uniformly distributed over L. If $2R$ is larger than L, then a single, stationary sensor in the middle of the barrier is sufficient. Otherwise, let the searcher's speed be V.

If θ is the angle between the two velocity vectors, then the searcher's speed relative to the tape is $S = \sqrt{U^2 + V^2 - 2UV\cos(\theta)}$. The relative speed S will vary with time because θ varies with time as the searcher moves around the closed curve. However, since the average value of $\cos(\theta)$ must be 0, because the curve is closed, the average value of S (symbolically $E(S)$) cannot exceed $\sqrt{U^2 + V^2}$. This follows from Jensen's inequality and the fact that the square root is a concave function.[28] Now, new tape area shows up at the rate UL, but the searcher cannot possibly examine it faster than the rate $2RS$ at any time or $2RE(S)$ on the average. The ratio of these two rates is, therefore, an upper bound on the detection probability:

$$P_D \le \frac{2RE(S)}{UL} \le \frac{2R}{L}\sqrt{1+\frac{V^2}{U^2}} \qquad ...(10)$$

Of course, P_D must also be smaller than 1. Referring to Figure 10, if the searcher were to move entirely in the East–West direction, his average speed relative to the tape, which is moving West at speed U, would be *V*, the same as if he never moved at all. If the searcher instead moves in the North–South direction, he achieves the preferred dynamically enhanced speed of $\sqrt{U^2+V^2}$, relative to the tape. The trouble is that he must reverse course when he nears the North or South border in order to keep his sensor on the tape, and this course reversal results in wastefully covering parts of the tape that have already been covered. Use of a figure of eight track, instead of a straight back-and-forth track, to some extent achieves a course reversal without double coverage, but only partially. Use of equation (10) amounts to the assumption that the searcher can find some way of achieving dynamic enhancement without suffering much from wasteful coverage of areas outside the tape or wasteful double coverage of areas on the tape.

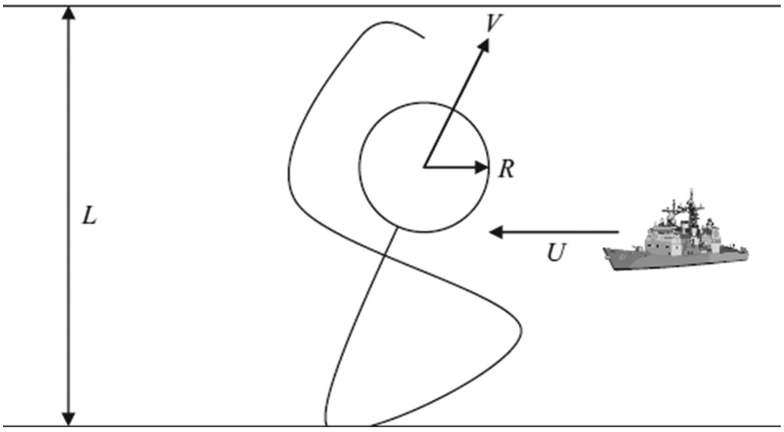

Figure 10 A Searcher Following a Figure of Eight Type Track at Speed U Hopes to Detect a Ship Moving at Speed V as it Tries to Penetrate a Barrier of Width L

Source: Washburn and Kress, *Combat Modelling*, n. 2.

Kinematic Enhancement vs Acoustic Degradation

Most of the things that a naval commander, tasked with search of a sub-surface submarine, wants to do can be done better at high speeds from a *kinematic* standpoint. However, an increase in searcher speed is accompanied by an increase in noise, in fact, two types of noise—self noise, which interferes with its listening ability, and radiated noise that discloses its presence to the adversary. They are denoted by L_N and L_s, respectively. They are measured in dB and are reflected additively in sonar figure of merit (FOM) as $L_s - L_N$. Thus, an increase in speed incurs acoustic degradation as well as kinematic enhancement. Trade-offs thereby arise in the tactical situation of a searcher patrolling a back and forth barrier versus a submarine transitor trying to penetrate the barrier undetected. For modelling purposes, first consider the forms of the relationships of self noise versus speed and radiated noise versus speed.[29]

At speeds below some 'breakpoint' speed, self noise is independent of speed. In this speed regime, self noise is governed by the ambient, electronic self noise and the speed-independent non-propulsion machinery necessary for the searcher's habitability functions. At speeds above this breakpoint, self noise is governed mainly by flow noise, from water passing the hull. It is assumed that the noise versus speed is linear in this regime. Thus, the relation between self noise and the speed is determined by three parameters:

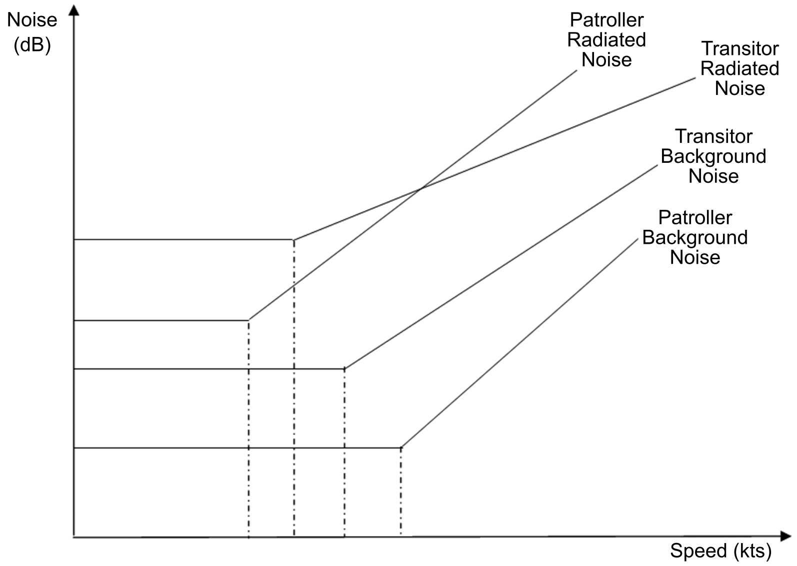

Figure 11 Illustrative Noise Speed Curves

Source: Wagner et al., *Operations Analysis Manual, Third Edition*, n. 1.

(a) The breakpoint speed, denoted by v_s.
(b) The constant self noise below the breakpoint, denoted by n_{sp}.
(c) The slope (in dB/knot) of the linear graph of self noise versus speed above the breakpoint, denoted by m_{sp}.

Similarly, for the evading target submarine, the parameters are denoted as u_s, n_{st} and m_{st}. These parameters will vary depending upon the platform. The same logic holds for the radiated noise as well. We define the same three parameters for the radiated noise and denote them as v_r, n_{rp} and m_{rp} for the searcher and u_r, n_{rt} and m_{rt} for the evading target.

As shown in Figure 11, we present the formulas for the four noise speed relations below for the searcher speed denoted by v and the evading target speed denoted by u:

Target self noise = $n_{st} + m_{st}\,(u - u_s)$,
Target radiated noise = $n_{rt} + m_{rt}\,(u - u_r)$,
Searcher self noise = $n_{sp} + m_{sp}\,(v - v_s)$,
Searcher radiated noise = $n_{rp} + m_{rp}\,(v - v_r)$,

The propagation loss N_W, needs to be further modelled in terms of range, r. This is considered to be a 'spreading law':

$$N_W = k \log_{10} r, \qquad \ldots(11)$$

where k is the 'spreading factor'. Hence,

$$r = 10^{N_W/k} \qquad \ldots(12)$$

For detection to occur, $N_W \leq FOM$. We know that

$$FOM = n_{rt} + m_{rt}\,(u - u_r) - n_{sp} - m_{sp}\,(v - v_s) + L = J + Z, \qquad \ldots(13)$$

where, $J = m_{rt}u - m_{sp}v$, and L and, hence, Z do not depend on v and u. Thus, the kinematic enhanced sweep width may be expressed as

$$Q10^{J/k}\sqrt{1+\frac{v^2}{u^2}} \equiv Qf\,(u,v) \qquad \ldots(14)$$

where Q embodies the effect of all components of FOM that do not depend on the speed of either unit. Although Q need not be computed, for purpose of derivation it is $2 \times 10^{Z/k}/d$.

Thus, f defined in equation (14) above, embodies both the kinematic and acoustic effects on sweep width that arise from the choices of speed by both the searcher and the evading target. Hence, f may be used as the pay off function in a game theory analysis of these speed choices.[30]

OPERATION ANALYSIS

The propagation of sound and the sonar, basic for a shallow water scenario, has been presented along with the search methodology. In this section, an attempt has been made to derive the random search solution for the Indian context using random search in a game theoretic approach. We carry forward the discussion on kinematic enhancement with acoustic degradation to present the complete solution. The first task is to give values to the variables in the realistic problem domain. Certain values have been given from available literature to solve the problem to show the methodology, however, it may be stated that realistic values could be fed from field data to derive solutions for a specific problem. The work has very high potential for exploitation in the field with realistic data.

We take spreading factor 'k' = 20. This is typically for deep waters and for shallow waters cylindrical spreading is used assuming specular reflections form the surface and the bottom. Spherical spreading is a very strong assumption of loss and, thus, we retain it to counter the error in the values due to the assumption of specular reflection as the sea surface and the bottom roughness will result in high levels of loss.

The speed of the searcher and the evading target is assumed to be above the break point speed as one would assume both to try and operate at high speeds to gain kinematic advantage. Further, we assume that both will restrict their speeds below cavitation (typically 20 knots for warships, including surface vessels and submarines) to ensure that the radiated noise is not high enough to be detected and identified at significant range from the adversary. This translates to searcher radiated noise slope m_{st} and evading target self noise slope m_{rp} remaining insignificant. Thus, speed range of 6, 8, 10, …, 20 knots have been used for both.

Four cases have been considered wherein the values of the searcher self noise slope m_{sp} and the evading target radiated noise slope m_{rt} have been varied between 0.1 dB/knot and 0.8 dB/knot, in some random steps. I now present the pay off matrices for the four cases. It may be noted that the pay off matrices will change for the specific realistic scenario. The objective here is to maximize the pay off for the target, i.e. columns, and minimize the pay off for the searcher, i.e. the rows.

Case 1: m_{sp} = 0.1 dB/knot and m_{rt} = 0.1 dB/knot

The pay off Table 1 suggests that the saddle point is achieved for $v = u =$ 20 knots. Thus, both the searcher and the evading target will maximize their benefits at their highest speeds under the given conditions. This translates into the fact that at low noise speed slopes, there is marginal acoustic degradation from high speeds and, thus, one can take maximum advantage from the kinematic enhancement.

Table 1: Illustrative Speed Choice Pay Off Matrix for m_{sp}=0.1 dB/knot and m_{rt}=0.1 dB/knot

v/u	*6*	*8*	*10*	*12*	*14*	*16*	*18*	*20*
6	1.41	1.28	1.22	1.20	1.19	1.20	1.21	1.23
8	1.63	1.41	1.31	1.26	1.23	1.23	1.23	1.24
10	1.86	1.56	1.41	1.33	1.29	1.26	1.25	1.25
12	2.03	1.72	1.53	1.41	1.35	1.31	1.29	1.28
14	2.32	1.88	1.64	1.50	1.41	1.36	1.33	1.31
16	2.54	2.04	1.76	1.59	1.48	1.41	1.37	1.34
18	2.75	2.19	1.88	1.68	1.56	1.47	1.41	1.38
20	2.96	2.35	1.99	1.77	1.63	1.53	1.46	1.41

Source: Wagner et al., *Operations Analysis Manual*, Third Edition, n. 1.

Case 2: m_{sp} = 0.8 dB/knot and m_{rt} = 0.8 dB/knot

The pay off Table 2 suggests that the saddle point is achieved for $v = u$= 6 knots. Thus, both the searcher and the evading target will maximize their benefits at their lowest speeds under the given conditions. This translates into the fact that at high noise speed slopes, there is significant acoustic degradation from high speeds and, thus, one can take minimum advantage from the kinematic enhancement.

Table 2: Illustrative Speed Choice Pay Off Matrix for m_{sp}=0.8 dB/knot and m_{rt}=0.8 dB/knot

v/u	*6*	*8*	*10*	*12*	*14*	*16*	*18*	*20*
6	1.41	1.50	1.69	1.94	2.27	2.68	3.18	3.79
8	1.39	1.41	1.54	1.74	2.00	2.34	2.75	3.25
10	1.34	1.33	1.41	1.56	1.78	2.05	2.39	2.81
12	1.29	1.25	1.30	1.41	1.58	1.81	2.09	2.44
14	1.22	1.16	1.19	1.50	1.41	1.60	1.83	2.12
16	1.13	1.07	1.09	1.15	1.26	1.41	1.61	1.85
18	1.05	0.98	0.99	1.04	1.13	1.25	1.41	1.62
20	0.96	0.89	0.89	0.93	1.00	1.11	1.24	1.41

Source: Wagner et al., *Operations Analysis Manual*, Third Edition, n. 1.

Case 3: m_{sp} = 0.5 dB/knot and m_{rt} = 0.7 dB/knot

The pay off Table 3 suggests that the saddle point is achieved for u = 8 knots and v = 10, 12, 14 knots. This is a case where the saddle point solution lies in between the extreme speeds for the target, i.e. 6 knots and 20 knots. The searcher can have three optimal points. However one may like to choose 14 knots being the highest to take maximum kinematic enhancement advantage.

Table 3: Illustrative Speed Choice Pay Off Matrix for m_{sp}=0.5 dB/knot and m_{rt}=0.7 dB/knot

v/u	*6*	*8*	*10*	*12*	*14*	*16*	*18*	*20*
6	1.62	1.69	1.85	2.08	2.38	2.75	3.18	3.70
8	1.71	1.70	1.81	1.99	2.25	2.56	2.95	3.41
10	1.77	1.72	1.78	1.93	2.14	2.41	2.74	3.15
12	1.82	1.72	1.75	1.86	2.04	2.27	2.57	2.93
14	1.84	1.72	1.72	1.81	1.95	2.16	2.41	2.73
16	1.84	1.70	1.68	1.75	1.87	2.04	2.27	2.56
18	1.82	1.66	1.64	1.68	1.79	1.94	2.14	2.39
20	1.78	1.62	1.58	1.62	1.70	1.84	2.02	2.24

Source: Wagner et al., *Operations Analysis Manual*, Third Edition, n. 1.

Case 4: m_{sp} = 0.5 dB/knot and m_{rt} = 0.6 dB/knot

Table 4: Illustrative Speed Choice Pay Off Matrix for m_{sp}=0.5 dB/knot and m_{rt}=0.6 dB/knot

v/u	*6*	*8*	*10*	*12*	*14*	*16*	*18*	*20*
6	1.52	1.54	1.65	1.81	2.03	2.28	2.59	2.94
8	1.59	1.55	1.61	1.74	1.91	2.13	2.39	2.71
10	1.65	1.56	1.59	1.68	1.82	2.00	2.23	2.50
12	1.70	1.57	1.56	1.62	1.74	1.89	2.09	2.33
14	1.72	1.56	1.53	1.57	1.66	1.79	1.96	2.17
16	1.72	1.55	1.50	1.52	1.59	1.70	1.85	2.03
18	1.70	1.52	1.46	1.47	1.52	1.61	1.74	1.90
20	1.67	1.48	1.41	1.41	1.45	1.53	1.64	1.78

Source: Wagner et al., *Operations Analysis Manual*, Third Edition, n. 1.

The pay off Table 4 suggests that here there is no saddle point. The alternative is to eliminate dominated rows and columns. The columns for u = 12, 14, 16, 18 and 20 knots are dominated by u = 10 knots. The rows for v = 16, 18 and 20 knots are dominated by v = 14 knots. These eliminations result in the pay off matrix shown below in Table 5.

Table 5: Reduced Speed Choice Pay Off Matrix for m_{sp}=0.5 dB/knot and m_{rt}=0.6 dB/knot

v/u	*6*	*8*	*10*
6	1.52	1.54	1.65
8	1.59	1.55	1.61
10	1.65	1.56	1.59
12	1.70	1.57	1.56
14	1.72	1.56	1.53
16	1.72	1.55	1.50

Source: Wagner et al., *Operations Analysis Manual*, Third Edition, n. 1.

Table 5 entries could be formulated into a two-person zero-sum game as a linear program for finding the optimal mixed strategy. The optimal pair of mixed strategies is given in Table 6.

Table 6: Optimal Solution for Mixed Strategy

Searcher Speed v (in knots)	*Apiori-probability of Searcher Speed v*	*Evading Target Speed u (in knots)*	*Apiori-probability of Target Speed u*
6	0.08	8	0.75
12	0.92	10	0.25

Source: Wagner et al., *Operations Analysis Manual, Third Edition*, n. 1.

The other speeds given in Table 5 have zero probability. The saddle point is v = 12 knots and u = 8 knots. Sometimes it is observed that deviation from the saddle point has marginal impact on the overall strategy. Thus, one may not stick to the saddle point very rigidly; however, it is important to know the deviations and its impact for the decision-maker.

The biggest challenge for any operations planner for the success of any search assignment is minimizing the uncertainties of the sensor performance. In this work, the uncertainties of the littoral waters have been enumerated in detail. It may be mentioned that the sonar commercially available are designed for deep waters where generalization is possible to freeze the design for some specified performance. However, in shallow waters, the site specific behaviour of the medium results in sub-optimal performance of the sensor at varying deployment locations. The way out of such a situation is massive oceanographic research to minimize the uncertainties of our own sonar and denying the same advantage to the adversary. This will improve the measure of performance of the sensor for the given search task. Further, having the right situational awareness of the medium condition will facilitate precise knowledge of the operations analysis formulation and, thus, the decision-maker will derive the optimality curve for the evolved strategy. Depending upon the optimality curve shape, the decision-maker will know the allowance for deviation and uncertainties of the medium. This formulation will enhance the overall measure of effectiveness of the operations analysis strategy. The measure of performance of the sensor and the measure of effectiveness of the entire strategy can be precisely ascertained with detailed understanding of the sonar performance, medium variability, and sensor deployment methodology.

Conclusions

In this article, an attempt has been made to put in perspective the medium uncertainties of littoral waters in the Indian subcontinent. The proposal is twofold—to develop a better understanding of the ocean medium by enhanced oceanographic studies in order to generate sufficient data for acoustic degradation due to environmental fluctuations. Such effort should create a predictive model that is able to generate the environmental data given the input of measurable physical parameters like instantaneous wind speed, sea surface temperature, sea state, bottom type, bottom profile, sound velocity profile, etc. The success of any operations analysis formulation is sensitive to the sensor performance; thus, predictive models will certainly help. However, alternately, availability of the precise data on the medium uncertainties can facilitate improved formulation of the operations analysis strategy to tackle the uncertainties and enhance the measure of effectiveness. In the operations analysis strategy presented in the work, the precise data on the acoustic degradation can facilitate improved generation of the optimality curve and understanding of the allowance for deviations due to medium uncertainties and other operational constraints. The IOR, and more specifically the Indian sub-continent, presents a significant challenge to any sonar designer and naval commander tasked with ASW operations due to its tropical waters in the littoral settings. Thus, a two-pronged tactical strategy for any ASW assignment, as proposed in this article, requires serious consideration.

The article only presents the method; however, the same can be extended to actual operational planning for a search assignment in the IOR with more specific data availability. The information used is from open source, as referenced, and no specific classified data has been accessed for any part of the report or formulation. The author may be accused of presenting significant amount of underwater signal processing theory; it may, however, be admitted that it was felt necessary to explain the complexity of the underwater signal propagation relevant to the operations analysis formulation of a search assignment. Further, the very relevant issue of sub-optimal performance of underwater systems in the Indian context has been addressed through this article.

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